CORRIGAN-FAIRLIE-'T HOOFT ANSATZ

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Abstract. In this note, we give an alternative treatment of the conformal ansatz for Yang-Mills instantons on Riemannian cones with special holonomy, which appear in various special cases elsewhere in the literature. This generalises the Gunaydin-Nicolai 't Hooft ansatz for Euclidian space in dimensions four, seven, and eight.

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1. Introduction

In the mathematical literature, the study of self-dual instanton solutions to the Yang-Mills equations on manifolds with special holonomy groups is motivated by the link to enumerative invariants [DT98], [DS11]. A number of recent works have focused on the case where the base manifold is a non-compact geometries asymptotic to a Riemannian cone [LO18, MNT22]. Explicit solutions can be produced, which can be seen as de-singularisations of solutions on the cone [ST23]. In particular, one is interested in solutions which interpolate between two distinct connections pulled back from the link of the cone.

On the physics side, generalisations of the 't Hooft ansatz for self-dual instantons on flat $\mathbb{R}^4$ have been studied at length, e.g. Gunaydin and Nicolai's solutions on $\mathbb{R}^7$ and $\mathbb{R}^8$, and generalisations have been made to special holonomy cones [HN12].

This note presents a unified treatment of the extension of this ansatz to Riemannian cones of holonomy G_2 and Spin(7), and clarifies the geometric interpretation of the connection that is produced.

1.1. ψ-instantons.

To fix conventions, let (M^n, g) be a Riemannian manifold of dimension n, and $E \to M$ be a vector bundle over M. Suppose, moreover, that (M, g) admits a parallel four form $\psi \in \Omega^4(M)$. There is an associated self-adjoint map $P_\psi : \Omega^2(M) \to \Omega^2(M)$, which is equivariant with respect to the holonomy group $\mathrm{Hol}(g)$ of g, defined by:

$$P_\psi(\alpha) := 2 * (\alpha \wedge *\psi) \qquad \alpha \in \Omega^2(M) \tag{1}$$

where $*$ is the Hodge star of g. A four-form ψ is said to be *admissible* for g if it is parallel, and the smallest eigenvalue of P_ψ is -2.

Suppose ψ is admissible. A connection form $\nabla \in \Omega^1(P, \mathrm{End}E)$ is said to be a ψ-instanton if its curvature two-form $F_\nabla \in \Omega^2(M, \mathrm{End}E)$ satisfies

$$*F_\nabla = -F_\nabla \wedge *\psi \tag{2}$$

i.e. the two-form part of its curvature lies in the -2 eigenspace of P_ψ. Note that ψ-instantons, if they exist, define absolute minima of the Yang-Mills energy on compact manifolds.

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In this note, we shall make three further assumptions that differ from the standard ones made elsewhere in the literature:

(H1) The -2-eigenspace of P_ψ can be identified pointwise with a lie-algebra $\mathfrak{g} \subset \mathfrak{so}(n)$ which contains the lie algebra of $\mathrm{Hol}(g)$.
(H2) The space of traceless symmetric tensors Sym^2_0 is $\mathfrak{g}$-irreducible at each point.
(H3) P_ψ has exactly one further non-zero eigenvalue $\lambda > -2$.

The examples we have in mind are when the lie algebra $\mathfrak{g}$ corresponds to the lie algebra of $SU(2)$, G_2, and $\mathrm{Spin}(7)$ in dimensions four, seven, and eight respectively. In the first case, we can take the four form ψ to be the volume form of a hyperkähler metric: then the eigenvalues ± 2 of P_ψ correspond to self- and anti self-dual two forms. In the G_2 and $\mathrm{Spin}(7)$ cases, we impose that the metric has one of these exceptional holonomy groups, and we can take ψ to be the stabiliser of its pointwise action on four-forms, suitably normalised. With this choice, the eigenvalue λ in assumption (H3) is 4 and 6 respectively.

The equation (2) is well-known in these three examples: in dimension four, for instance, (2) is the classical anti-self-dual equations; in higher dimensions (2) is referred to as the G_2-*instanton* and $\mathrm{Spin}(7)$-*instanton* equations respectively [FSE19, §2].

Remark 1. Other kinds of instantons, like the Hermitian Yang-Mills equation, can be viewed in the set-up of (2). However, assumption (H3) excludes them from using the ansatz presented here.

1.2. **Main Results.** The curvature tensor of the Levi-Civita connection of any Riemannian metric can be identified with a symmetric map on two-forms. In particular, with assumption (H1), the Levi-Cevita connection of a Riemannian manifold (M, g) with an admissible ψ is itself a solution of (2) on the tangent bundle TM. In this note, we give an example of another solution of (2) on the tangent bundle, which recovers the 't Hooft ansatz for flat $\mathbb{R}^4$, generalised by Gunaydin–Nicolai to $\mathbb{R}^7$ and $\mathbb{R}^8$.

Theorem 1. *Suppose (M^n, g) is a Riemannian manifold with an admissible $\psi \in \Omega^4(M)$, satisfying assumptions (H1), (H2) and (H3). If (M, g) admits a non-constant solution h to*

$$\mathrm{Hess}(h) = \tfrac{1}{n}(\Delta h)g \tag{3}$$

Then TM admits a ψ-instanton on TM via the construction in §2.

In particular, this theorem applies to anti-self-dual instantons on hyperkähler 4-manifolds, G_2-instantons on G_2-manifolds, and $\mathrm{Spin}(7)$-instantons on $\mathrm{Spin}(7)$-manifolds.

However, in the cases indicated above, this is rather restrictive. We have the following proposition, which follows from [Pet72, Theorem 4.3.3], and [Bes07, §9.109]:

Proposition 1. *Suppose (M, g) is a Riemannian manifold as in Theorem, and suppose $\mathrm{Ric}(g) = 0$. Then (M, g) admits a non-constant solution of (3) if and only if (M, g) is a Riemmannian cone.*

Recall that a Riemannian cone is a manifold $N \times (0, \infty)$ equipped with a metric $g = dr^2 + r^2 g_N$ where (N, g_N) denotes some fixed Riemannian manifold, and $r \in (0, \infty)$ is the radial coordinate. The solution h of Proposition 1 can be given uniquely as

$$h = r^2 + 1 \tag{4}$$

up to an overall scaling of the metric $g \mapsto \varepsilon g$ for some $\varepsilon > 0$, and shift by a constant $h \mapsto h + c$.

Remark 2. Note that [EH01, Proposition 3.1] claims that no solutions exist to (3) if g is Ricci-flat and holonomy irreducible. However, in dimension $n > 4$ there are many holonomy irreducible Ricci-flat cones e.g. the Calabi-Yau metric on the conifold [CdlO90], or Bryant's exceptional holonomy examples [Bry87]: thus (4) gives many counterexamples to this claim. We are unable to determine the source of the discrepancy in [EH01, Proposition 3.1], however, they do not appear to use completeness in their proof.

Remark 3. The connection on (M, g) used in the construction of §2 is related to the Levi-Civita connection of a conformally rescaled metric, but the the precise conformal factor has no obvious geometric significance outside of the anti-self dual setting.

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AI Usage Statement. No LLMs were used in the making of this note.

2. Construction

Let (M, g) be a Riemannian manifold with an admissible four form ψ, satisfying assumptions (H1), (H2) and (H3). Given some differentiable function f on M, and a vector field X on (M, g), let σ_X be the skew-adjoint endomorphism given by:

$$\sigma_X(Y) := Y(f)X - g(X, Y)\nabla f \tag{5}$$

Since this is linear in X, this defines a one-form with values in skew-adjoint endomorphisms, which we will denote σ.

Remark 4. σ_X is the just skew-adjoint part of the difference $\tilde{\nabla}_X - \nabla_X$ of the Levi-Civita connections of g and the conformally rescaled metric $\tilde{g} = e^{2f}g$.

Now, using the operator P_ψ, denote π as the map on two-forms

$$\pi(\alpha) := \tfrac{1}{2}(\alpha - \tfrac{1}{\lambda}P_\psi(\alpha)) \qquad\qquad \alpha \in \Omega^2(M) \tag{6}$$

which we will also identify with a corresponding map on skew-adjoint endomorphisms. Pointwise, the image of this map is contained in $\mathfrak{g}$ of (H1), and differs from the projection to the -2 eigenspace of P_ψ by a factor of $\frac{\lambda+2}{2\lambda}$ by assumption (H3).

Remark 5. In particular, it in case of the anti self-dual instantons $\lambda = 2$, and π *is* the projection map, but it differs in general.

Using (6), we define the connection on TM given by

$$\nabla' := \nabla + \pi(\sigma) \tag{7}$$

where ∇ is the Levi-Civita connection of g.

Let $F = F_{\nabla'}$ denote the corresponding curvature operator. Since the Levi-Civita connection has curvature that takes values in $\mathfrak{g}$ by assumption (H1), then F is also a two-form that takes values in $\mathfrak{g}$. By identifying the space of two-forms with the space of skew-adjoint endomorphisms, F also defines an endomorphism on $\Omega^2(M)$. We can ensure ∇' is a ψ-instanton (2) if this endomorphism is symmetric.

Theorem 2. *Let $h := e^{-f}$, and F be the curvature of the connection ∇' in (7). Then F defines a symmetric map on $\Omega^2(M)$ if and only if the traceless Hessian of h vanishes, i.e.*

$$\mathrm{Hess}(h) = \tfrac{1}{n}(\Delta h)g \tag{8}$$

In particular, if (8) holds, then ∇' is a ψ-instanton.

Proof. We compute in an orthonormal frame e_i, summing over repeated indices. We use the convention $e^{ij} := e^i \wedge e^j$, where e^i denotes the elements of the orthonormal co-frame dual to e_i. If $A = \frac{1}{2}A_{ij}e^{ij}$ denotes some two-form, the two form $P_\psi(A) = \frac{1}{2}\left[P_\psi(A)\right]_{ij}e^{ij}$ has entries given by

$$\left[P_\psi(A)\right]_{ij} = \psi_{ijkl}A_{kl}.$$

where ψ_{ijkl} are the coefficients of the four form ψ in this frame, which we can assume to be constant.

Denote the connection one-forms of ∇, ∇' in this frame by α^{ij}, $(\alpha')^{ij}$. Then $\nabla e^j = e^i \otimes \alpha^{ij}$ and $\sigma^{ij} = -\nabla f \lrcorner e^{ij}$. So

$$(\alpha')^{ij} = \alpha^{ij} + [\pi(\sigma)]^{ij} = \alpha^{ij} - \tfrac{1}{2}\nabla f \lrcorner (e^{ij}) + \tfrac{1}{\lambda}\psi_{ijkl}e_k(f)e^l.$$

Our assumptions ensure that pointwise, both α^{ij} and $\pi(\sigma^{ij})$ are entries of a matrix lying in the lie algebra $\mathfrak{g}$ with one-form coefficients.

Let R^{ij} denote the Levi-Civita curvature operator of g. Since $F^{ij} = d\,(\alpha')^{ij} + (\alpha')^{ik} \wedge (\alpha')^{kj}$ and π is parallel for the Levi-Civita connection, then we have:

$$\begin{aligned} F^{ij} &= R^{ij} + \left(+d[\pi(\sigma)]^{ij} + \alpha^{ik} \wedge [\pi(\sigma)]^{kj} + [\pi(\sigma)]^{ik} \wedge \alpha^{kj}\right) + \pi(\sigma)^{ik} \wedge \pi(\sigma)^{kj} \\ &= R^{ij} + \left[\pi\left(d\sigma + \alpha \wedge \sigma + \sigma \wedge \alpha\right)\right]^{ij} + [\pi(\sigma)]^{ik} \wedge [\pi(\sigma)]^{kj} \end{aligned}$$

Since π only acts on the endomorphism part of the terms in the above expression, let us first evaluate the second term in the expression for F^{ij} on vector-fields e_l, e_m:

$$\begin{aligned} \left[d\sigma^{ij} + \alpha^{ik} \wedge \sigma^{kj} + \sigma^{ik} \wedge \alpha^{kj}\right](e_l, e_m) &= \left[\mathrm{Hess}(f)_{kj}e^{ki} - \mathrm{Hess}(f)_{ik}e^{kj}\right](e_l, e_m) \\ &= \mathrm{Hess}(f)_{kp}\left(\delta^p_j(\delta^k_l\delta^i_m - \delta^k_m\delta^i_l) - \delta^p_i(\delta^k_l\delta^j_m - \delta^k_m\delta^j_l)\right) \end{aligned}$$

We also compute:

$$\begin{aligned}\left[\pi(\sigma^{ik})\wedge\pi(\sigma^{kj})\right](e_l,e_m) &= \pi\left(\delta^k_l e_i(f)-\delta^i_l e_k(f)\right)\pi\left(\delta^j_m e_k(f)-\delta^k_m e_j(f)\right)\\ &\quad-\pi\left(\delta^j_l e_k(f)-\delta^k_l e_j(f)\right)\pi\left(\delta^k_m e_i(f)-\delta^i_m e_k(f)\right)\end{aligned}$$

where π in this context appears as e.g. acting on the k,i-indices of $\delta^k_l e_i(f)-\delta^i_l e_k(f)$ to get:

$$\pi\left(\delta^k_l e_i(f)-\delta^i_l e_k(f)\right)=\tfrac{1}{2}\left(\delta^k_l e_i(f)-\delta^i_l e_k(f)\right)-\tfrac{1}{\lambda}\psi_{ikpq}\delta^q_l e_p(f)$$

We wish to determine if the curvature tensor satisfies $F^{ij}_{kl}=F^{kl}_{ij}$, where we define $F^{ij}_{kl}:=F^{ij}(e_k,e_l)$, so that $F^{ij}=\frac{1}{2}F^{ij}_{kl}e^{kl}$. One observes[1] that the expression $F^{ij}_{kl}-F^{kl}_{ij}$ will only have terms involving ψ. Up to ψ-independent terms, we have:

$$\begin{aligned}F^{ij}_{lm} &= \tfrac{1}{\lambda^2}\left(\psi_{lsjk}\psi_{mqki}-\psi_{lqki}\psi_{msjk}\right)e_q(f)e_s(f)\\ &\quad+\tfrac{1}{\lambda}e_p(f)\left(\psi_{iplm}e_j(f)-\psi_{jplm}e_i(f)\right)-\tfrac{1}{\lambda}\left(\psi_{ijmp}\mathrm{Hess}(f)_{lp}-\psi_{ijlp}\mathrm{Hess}(f)_{mp}\right)+\text{l.o.t.}\end{aligned}$$

The $O(\psi^2)$-term automatically disappears from $F^{ij}_{lm}-F^{lm}_{ij}$, so we are left with:

$$\lambda(F^{ij}_{lm}-F^{lm}_{ij})=S_{jp}\psi_{iplm}+S_{ip}\psi_{pjlm}-S_{lp}\psi_{ijpm}-S_{mp}\psi_{ijlp}$$

where S_{ij} is defined by:

$$S_{ij}:=e_i(f)e_j(f)-\mathrm{Hess}(f)_{ij}$$

This says that $F\in\mathrm{Sym}^2\left(\Omega^2(M)\right)$ if and only if P_ψ commutes with $S\diamond$ commutes as a map on $\Omega^2(M)$, where S is the symmetric tensor defined by

$$S=df\otimes df-\mathrm{Hess}(f)=e^f\mathrm{Hess}(e^{-f}) \tag{9}$$

and $S\diamond$ is the diamond action of S on $\Omega^2(M)$, see appendix A. However, by assumption (H2), and Proposition 3 of the appendix, P_ψ and $S\diamond$ commute if and only if S is a multiple of the identity. Hence, this gives that the trace-free part of the symmetric tensor $\mathrm{Hess}(e^{-f})$ vanishes. □

Suppose (N,g_N) denotes some Riemannian manifold such that the cone metric $g=dr^2+r^2g_N$ on $\mathbb{R}_{>0}\times N$. Then we have the following proposition:

Proposition 2. *Let $\mathbb{R}_{>0}\times N$ be equipped with the cone metric $dr^2+r^2g_N$. There are a two-parameter family (C_1,C_2) of solutions to (8) given by*

$$h(r)=C_1+C_2r^2 \tag{10}$$

Proof. Let $h=h(r)$ be constant along N. For any pair of vector-fields X,Y pulled back from N, and the radial vector field ∂_r:

$$\mathrm{Hess}(f)(\partial_r,\partial_r)=f''\qquad\mathrm{Hess}(f)(\partial_r,X)=0\qquad\mathrm{Hess}(f)(X,Y)=rg_N(X,Y).$$

Then (8) gives the ODE:

$$h''=rh' \tag{11}$$

which is integrable, and has the two-parameter family of solutions as above. □

Since the connection ansatz ∇' in Theorem 2 depends only on the derivative ∇h, applying Propostion 1 to Theorem 2 gives gives a one-parameter family of ψ-instantons on any Riemannian cone with holonomy contained in $SU(2)$, G_2, or Spin(7). This reproduces the 't Hooft ansatz on $\mathbb{R}^4$, $\mathbb{R}^7$ and $\mathbb{R}^8$ as the cone over the round sphere of appropriate dimension, and gives non-trivial solutions on the holonomy G_2, and Spin(7) cones of Bryant [Bry87]. We can account for the second-parameter by pulling back via the rescaling $r\mapsto\varepsilon r$ for some $\varepsilon>0$, which rescales the metric by an overall constant.

Remark 6. Despite the close relationship with the geometry of the round sphere, the connection ∇' on Euclidean space in dimensions 7, 8 does not appear directly related to the Levi-Civita connection of the round metric: it appears with a different power of the conformal factor (10) cf. Remark 4. Nor, in the case of S^7, is it related to the canonical connection with totally skew torsion

[1] Either directly, or as a consequence of the fact σ is (the anti-symmetric part of) the difference of two Levi-Civita connections, which always satisfy $R^{ij}_{kl}=R^{kl}_{ij}$ by the Bianchi indentity, cf. Remark 4, also [Tau11, §16.6].

associated to the so-called *nearly parallel* G_2-structure. One can compute that the torsion of ∇' is given by:

$$T_{\nabla'}(X,Y) = \tfrac{1}{2}(X(f)Y - Y(f)X) + \tfrac{2}{\lambda}\psi(\nabla f, X, Y, \cdot)^\sharp$$

where $\sharp : T^*M \to TM$ is the musical isomorphism.

3. Further questions

- Can this construction be generalised to non-integrable G_2 or Spin(7) structures? In particular, can we use a similar ansatz starting with the nearly-parallel instanton on S^7?
- Given any non-integrable G_2 or Spin(7)-structure, can we always find a non-torsion free G_2-holonomy connection whose curvature F is a symmetric operator $F \in \mathrm{Sym}\left(\Omega^2(M)\right)$?
- Does this ansatz work for other holonomy groups? E.g. $\mathrm{Sp}(n)\mathrm{Sp}(1)$.

Appendix A. Linear algebra

Let V be a vector-space, with dual V^*. We will extend the standard interior product between vectors and forms $\lrcorner : V \otimes \Lambda^\bullet(V^*) \to \Lambda^\bullet(V^*)$ to a product on $\diamond : \mathrm{End}(V) \otimes \Lambda^\bullet(V^*) \to \Lambda^\bullet(V^*)$, cf. [KLL20]:

Definition 1. Let $S \in \mathrm{End}(V)$, $\alpha \in \Lambda^k(V^*)$, then $S \diamond \alpha \in \Lambda^k(V^*)$ is defined such that, for any $X_1, \dots, X_k \in V$:

$$(S \diamond \alpha)(X_1, \dots X_k) = \sum_{i=1}^{k} \alpha(X_1, \dots, S(X_i), \dots, X_k)$$

In other words $S \diamond \alpha := \frac{d}{dt}\big|_{t=0} (\exp(tS)^*\alpha)$.

Suppose V is equipped with an inner-product g, then we have a splitting of the space of linear maps on V into symmetric and anti-symmetric parts:

$$\mathrm{End}(V) \cong \mathrm{Sym}^2(V^*) \oplus \Lambda^2(V^*)$$

which is orthogonal with respect to the induced inner-product on $\mathrm{End}(V)$, and equivariant with respect to the natural $\Lambda^2(V^*)$-action on $\mathrm{End}(V)$ given by the Lie bracket. Here, we are identifying the two summands on the right with $\mathrm{End}(V)$ in the standard way i.e. identify $A \in \mathrm{End}(V)$ with $A^\flat \in V^* \otimes V^*$ such that $g(AX, Y) = A^\flat(X, Y)$ for any X, Y in V.

With this identification, we shall write, for any $S \in \mathrm{End}(V)$, $A \in \Lambda^2(V^*)$:

$$S \diamond A := AS + S^t A \tag{12}$$

where S^t is the transpose of S with respect to g. Note that the transpose of the map $S\diamond \in \mathrm{End}(\Lambda^2(V^*))$ is just $S^t \diamond A$.

Lemma 1. *With respect to the natural $\Lambda^2(V^*)$-action, $\diamond$ gives an embedding of $\Lambda^2(V^*)$-representations*

$$\diamond : \mathrm{End}(V) \hookrightarrow \mathrm{End}(\Lambda^2(V^*))$$

Proof. The discussion above shows that $\diamond$ respects the splitting into symmetric and anti-symmetric parts. The claim this is equivariant follows from

$$[A, S] \diamond B = [A, S \diamond B] - S \diamond ([A, B]) \tag{13}$$

for $A, B \in \Lambda^2(V^*)$.

The injectivity follows by considering the resulting splitting into irreducible representations in $\mathrm{End}(V)$. □

Now suppose V is finite dimensional, and we are given some four form $\psi \in \Lambda^4(V^*)$. This gives rise to a bilinear form on $\Lambda^2(V^*)$ by the inclusion $\Lambda^4(V^*) \subset \mathrm{Sym}^2(\Lambda^2(V^*))$, which we can view as a symmetric linear map $P_\psi \in \mathrm{End}(\Lambda^2(V^*))$ using the metric. In other words, for any skew-symmetric $A \in \mathrm{End}(V)$, $P_\psi(A)$ is defined such that:

$$g(P_\psi(A)X, Y) := \sum_i \psi(A(e_i), e_i, X, Y)$$

where $X, Y \in V$, and we sum over some orthonormal basis e_i for V.

Proposition 3. *Suppose there is a lie algebra $\mathfrak{g} \subset \Lambda^2(V^*)$ such that the space of traceless symmetric linear maps $\mathrm{Sym}^2_0(V)$ form an irreducible representation of $\mathfrak{g}$, and P_ψ is $\mathfrak{g}$-equivariant and injective. Then S and P_ψ commute if and only if S is a constant multiple of the identity.*

Proof. Since P_ψ is equivariant, with trivial kernel, then both P_ψ and the map $S\diamond$ must act as multiples of the identity on irreducible $\mathfrak{g}$-representations in $\Lambda^2(V^*)$. However, we have $S \in \mathrm{Sym}^2_0(V) \oplus \mathbb{R}I$ is a splitting into $\mathfrak{g}$-irreducible components, where I denotes the identity map, so by (13) the only possibility is that $S = \lambda I$. □

Note that we are identifying the inner-product g with the identity endomorphism I by raising/lowering indices as appropriate.

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